The GFDL Finite-Volume Cubed-sphere Dynamical Core

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What is FV$^3$? FV$^3$ is:

- **Fully finite volume!** Flux divergences + vertical Lagrangian + integrated PGF

- **Mimetic:** Recovers Newton’s and conservation laws with integral theorems

- **Adaptable and Robust:** works with many physics and chemistry packages! AM2/3/4, GOCART, MOZART, CAM, GFS, GEOS, etc. Also excellent for ocean coupling

- **Flexible:** arbitrary vertical levels, grid refinement by nesting and/or stretching

- **Fast!** A faster model tends to be a better model

- **Proven effective at all scales.** Maintains the large-scale circulation while accurately representing mesoscale and cloud-scale
Figure 10 shows the corresponding results for the west Pacific and North Atlantic in the observations is comparable in model and observations after the nor-
tropical storms. IBTrACS observations are shown by black line and circles. Four
Figure 2.

Hovmöller diagrams of daily surface precipitation (unit: mm day

Chen and Lin: TC Seasonal Prediction with GFDL HiRAM
Who uses FV$^3$?

- FV and FV$^3$ are among the most widely used global cores in the world, with a large and diverse community of users.

- GFDL models
  - AM4/CM4/ESM4
  - HiRAM
  - CM2.5/2.6
  - FLOR and HiFLOR
  - fvGFS

- CAM-FV$^3$ (FV is default in CESM)
  - LASG FAMIL
  - NASA GEOS
  - Harvard GEOS-CHEM
  - GISS ModelE
  - MPI ECHAM (advection scheme)
  - JAMSTEC MIROC (adv. scheme)
Development of the FV$^3$ core

- Lin and Rood (1997, QJ): FV solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Putman and Lin (2007, JCP): Cubed-sphere solver
- Lin (in prep): Nonhydrostatic dynamics
- Harris and Lin (2013) and Harris, Lin, and Tu (2016): Grid refinement
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Lin and Rood (1996, MWR)
Flux-form advection scheme

\[ q^{n+1} = \frac{1}{\pi_{n+1}^{+1}} \left\{ \pi^n q^n + F \left[ q^n + \frac{1}{2} g(q^n) \right] + G \left[ q^n + \frac{1}{2} f(q^n) \right] \right\} . \]

- **Forward-in-time** 2D scheme derived from 1D PPM operators
- Advective-form inner operators \((f, g)\) eliminate leading-order deformation error
  - Allows preservation of constant tracer field under nondivergent flow
- Ensures forward-in-time scheme is stable
- **Fully 2D**! Stability condition is \(\max(C_x, C_y) < 1\)
- Flux-form outer operators \(F, G\) ensure mass conservation
Lin and Rood (1996, MWR)
Flux-form advection scheme

\[ q^{n+1} = \frac{1}{\pi^{n+1}} \left\{ \pi^n q^n + F \left[ q^n + \frac{1}{2} g(q^n) \right] + G \left[ q^n + \frac{1}{2} f(q^n) \right] \right\} \]

• PPM operators are upwind biased
  - More physical, but also more diffusive

• Monotonicity constraint to prevent extrema; also option for “linear” (unlimited) non-monotonic scheme. Tracer advection is always monotonic.

• Scheme maintains linear correlations between tracers when unlimited or when monotonicity constraint applied (not necessarily so for positivity)
1D Advection Test

Lin and Rood 1996, MWR
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FV solver

- Solves adiabatic layer-averaged vector-invariant equations. $\delta p$ is the layer mass.

- Everything (except the PGF) is a flux! So we use the Lin & Rood advection scheme for forward evaluation.

- PGF evaluated backward with updated pressure and height

**Question:** how is vertical transport incorporated?
Lin and Rood (1997, QJ)
FV solver

- D-grid, with C-grid winds for fluxes
  - C-grid winds advanced a half-timestep—like a simplified Riemann solver. Diffusion due to C-grid averaging is alleviated

- Two-grid discretization and time-centered fluxes avoid computational modes
  - Divergence is invisible to solver: divergence damping is an integral part of the solver
FV solver: Vorticity flux

- Nonlinear vorticity flux term in momentum equation, confounding linear analyses
- D-grid allows exact computation of absolute vorticity—no averaging!
- Vorticity uses same flux as $\delta p$: consistency improves geostrophic balance, and SW-PV advected as a scalar!
- **Many** flows are strongly vortical, not just large-scale…

*Figure 10. Polar stereographic projections (from the sphere in the north pole) of the potential vorticity contours at DAY 24 in the ‘mesospheric vortexgenesis’ test case at three different resolutions.*
FV solver:
Kinetic Energy Gradient

- Vector-invariant equations susceptible to Hollingsworth-Kallberg instability if KE gradient not consistent with vorticity flux

- Solution: use C-grid fluxes again to advect wind components, yielding an upstream-biased kinetic energy

\[ \kappa^* = \frac{1}{2} \left\{ \mathcal{X}(u^\theta, \Delta t; u^n) + \mathcal{Y}(v^\lambda, \Delta t; v^n) \right\} \]

- Consistent advection again!
Development of the FV$^3$ core

• Lin and Rood (1996, MWR): Flux-form advection scheme

• Lin and Rood (1997, QJ): FV solver

• **Lin (1997, QJ): FV Pressure Gradient Force**

• Lin (2004, MWR): Vertically-Lagrangian discretization

• Putman and Lin (2007, JCP): Cubed-sphere solver

• Lin (in prep): Nonhydrostatic dynamics

• Harris and Lin (2013) and Harris, Lin, and Tu (2016): Grid refinement
Lin (1997, QJ)

Finite-Volume Pressure Gradient Force

- Computed from Newton’s second and third laws, and Green’s Theorem
- Errors lower, with much less noise, compared to a finite-difference pressure gradient evaluation
- Easily carries over to nonhydrostatic solver
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Vertically-Lagrangian Discretization

• Equations of motion are vertically integrated to yield a series of layers, which deform freely during the integration

• **Truly Lagrangian!** All flow follows the Lagrangian surfaces, including vertical motion. Vertical transport is *entirely* implicit, so…

  • **No** vertical Courant number restriction!! This is *critical* for high vertical resolution in the boundary layer

• To avoid layers from becoming infinitesimally thin, vertical remapping to “Eulerian” layers is periodically performed
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Putman and Lin (2007, JCP)
Cubed-sphere solver

- Gnomonic cubed-sphere grid: coordinates are great circles
- Widest cell only $\sqrt{2}$ wider than narrowest
  - More uniform than conformal, elliptic, or spring-dynamics cubed spheres
- Tradeoff: coordinate is non-orthogonal, and special handling needs to be done at the edges and corners.

![Diagram of wind staggerings and fluxes for a cell on a non-orthogonal grid. The angle is that between the covariant and contravariant components; in orthogonal coordinates $\alpha = \theta/2$.](image)
Non-orthogonal coordinate

- Gnomonic cubed-sphere is non-orthogonal

- Instead of using numerous metric terms, use covariant and contravariant winds
  - Solution winds are covariant, advection is by contravariant winds
  - KE is product of the two
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Nonhydrostatic FV³

- **Goal**: Maintain hydrostatic circulation, while accurately representing non-hydrostatic motions in the fully-compressible Euler equations

- Introduce new prognostic variables: $w$ and $\delta z$ (height thickness of a layer), from which density (and thereby nonhydrostatic pressure) is computed

- Traditional semi-implicit solver for handling fast acoustic waves

  - **True nonhydrostatic!** Explicit $w$ into vertically-Lagrangian solver

- Vertical velocity $w$ is the 3D cell-mean value. Vorticity is also a cell-mean value, so **helicity** can be computed without averaging!
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Stretched grid
The simple, easy way to achieve grid refinement

• Smooth deformation! And requires no changes to the solver

• Smooth grid has no abrupt discontinuity, and greatly reduces need for scale-aware physics

• Capable of extreme refinement (80x!!) for easy storm-scale simulations on a full-size earth

Harris, Lin, and Tu, 2016
Two-way grid nesting

• **Simultaneous** coupled, consistent global and regional solution. No waiting for a regional prediction!

• Different grids permit different parameterizations; **doesn’t need a “compromise” or scale-aware physics** for high-resolution region

• Coarse grid can use a longer timestep: **more efficient** than stretching!

• **Very flexible!** Combine with stretching for very high levels of refinement

Harris and Lin, 2013, 2014
FV solver:
Time-stepping procedure

- Interpolate time $t^n$ D-grid winds to C-grid
- Advance C-grid winds by one-half timestep to time $t^{n+1/2}$
- Use time-averaged air mass fluxes to update $\delta p$ and $\theta_v$ to time $t^{n+1}$
- Compute vorticity flux and KE gradient to update D-grid winds to time $t^{n+1}$
- Use time $t^{n+1}$ $\delta p$ and $\theta_v$ to compute PGF to complete D-grid wind update
**FV³ nonhydrostatic** solver: Time-stepping procedure

- Interpolate time $t^n$ D-grid winds to C-grid

- Advance C-grid winds by one-half timestep to time $t^{n+1/2}$

- Use time-averaged air mass fluxes to update $\delta p$ and $\theta_v$, \textbf{and to advect $w$ and $\delta z$, to time $t^{n+1}$}

- Compute vorticity flux and KE gradient to update D-grid winds to time $t^{n+1}$

- \textbf{Solve nonhydrostatic terms for $w$ and nonhydrostatic pressure perturbation using vertical semi-implicit solver}

- Use time $t^{n+1} \delta p$, $\delta z$, and $\theta_v$ to compute PGF to complete D-grid wind update
Mass conserving two-way nesting

- Usually quite complicated: requires flux BCs, conserving updates, and precisely-aligned grids

- Update only winds and temperature; not $\delta p$, $\delta z$, or tracer mass

  - Two-way nesting overspecifies solution anyway

- **Very simple**: works regardless of BC and grid alignment

  ★ $\delta p$ is the vertical coordinate: need to remap the nested-grid data to the coarse-grid’s vertical coordinate

- Option: “renormalization-conserving” tracer update